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LETTER TO THE EDITOR

Density of states of a two-dimensional electron system in a transverse magnetic field with a random potential

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Abstract. We consider the problem of a two-dimensional disordered electron system in a perpendicular applied magnetic field. Using the method of Feynman path integration and a Gaussian model of disorder, the density of states is calculated exactly within the first cumulant approximation and good comparison with magneto-optical experiments is obtained.

The problem of electrons confined in two dimensions under the influence of a transverse magnetic field and a random potential is now of theoretical interest. Examples of two-dimensional electron systems (2DES) are electrons confined to metal-oxide–semiconductor (MOS) space charge layers and at semiconductor heterojunctions. Earliest interest was in electron transport properties, the integer quantum Hall effect [1, 2] (QHE) and fractional quantum Hall effects, [1, 3] and in distinguishing localized from delocalized states. In the QHE, von Klitzing *et al* [4] showed that the conductivity in the plane, σ_{xy}, rather than being a simple linear function of the magnetic field, had plateaus of constant value, $\sigma_{xy} = (e^2/h)i$, where i is an integer ($i = \ldots, 3, 2, 1$). Now, it is believed that the plateaus in σ_{xy} occur when the Fermi energy E_F passes between the Landau levels (LLs) and through localized states, which do not contribute to the conductivity. Recent determinations of the electron states at the Fermi level by measuring the oscillatory dependence of the specific heat [5–8], capacitance [9, 10], magnetization [11, 12] and recombination spectra [13] of a two-dimensional electron system show that the disorder, due to impurities [14] or to inhomogeneities [15], broadens the LLs significantly. They confirm that there is a large density of states (DOS) between LLs.

The DOS of an electron confined in two dimensions in the presence of a transverse magnetic field and a disorder potential may be evaluated using several techniques developed for disordered systems. In a Born approximation [1, 9, 14, 16], a perturbative approach, the DOS is elliptical around each LL and zero between LLs. More exact methods [17–21] yield a Gaussian DOS for the lowest LL, as do path-integral methods [22, 23]. Broderix *et al* [24] discuss all these methods and the approximations in them carefully.

In a previous paper [23] it was shown that a substantial DOS between LLs could be obtained using non-perturbative methods for electrons interacting with disorder having a finite correlation length L. In the model considered, the disorder was represented by

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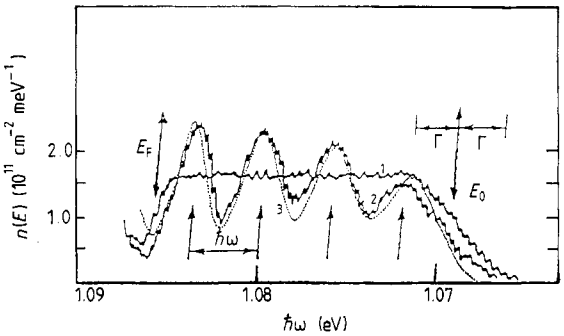

Figure 1. Curves 1 and 2 show the emission spectra of 2D electrons found for $T = 1.6$ K with $B = 0$ (spectrum 1) and $B = 7$ T, $\hbar\omega = 4$ meV (spectrum 2). Curve 3 (broken curve) shows the numerical result result for the DOS using $\xi_L = 6.8\ \text{meV}^2$ with $L = 97$ Å. The magnitude of the DOS at $B = 0$, $n_0 = 1.6 \times 1011\ \text{cm}^{-2}\ \text{meV}^{-1}$, was found by equating the integrated emission intensities in spectra 1 and 2.

the variance $W(\boldsymbol{r} - \boldsymbol{r}') = \langle V(\boldsymbol{r})V(\boldsymbol{r}')\rangle$ of the disorder potential $V(\boldsymbol{r})$ seen by the electrons. The variance was modelled by a Gaussian

$$W(\boldsymbol{r} - \boldsymbol{r}') = \xi_L \exp[-(\boldsymbol{r} - \boldsymbol{r}')^2/L^2] \tag{1}$$

in which the correlation length L and coefficient ξ_L are parameters. By making a large-T (time) approximation, the DOS is a sum of Gaussians centred at each Landau level E_n and it was found that the observed broadening [12] can be reproduced with $L \simeq 100$ Å and $\xi_L \simeq 80\ \text{meV}^2$. The origin of the disorder was not specified in the model, but the LL broadening is generally believed to be due to impurities [1, 16].

In this paper we calculate, without the large-T approximation, the DOS exactly within the first cumulant approximation using numerical integration. We show that the first cumulant approximation is sufficient to obtain an appropriate DOS which compares well with experiments [13]. It is known that there are various methods used to determine the DOS. However, all experiments may be classified into two different methods, the thermally activated magneto-conductances [5–12] and the magneto-optic [13]. For our investigations, we compare our numerical results with the magneto-optic experiments of Kukushkin and Timofeev (see figure 1) which now seem to be the most direct method to determine the 2D-electron density of states.

An electron confined in the interfaces of a MOS inversion layer is an example of a particle in a disordered environment. The DOS of the electrons may be evaluated using several techniques developed for disordered systems. We use here a path-integral technique developed previously for electrons in disordered, bulk materials [25, 26]. We consider a single electron interacting with a disorder potential

$$V(\boldsymbol{r}) = \sum_i v(\boldsymbol{r} - \boldsymbol{R}_i)$$

where $v(\boldsymbol{r})$ is an electron–impurity potential. The impurities are randomly located at $\boldsymbol{R}_i$ and create fluctuations in $V(\boldsymbol{r})$. The Hamiltonian for this single electron in the x–y plane and a perpendicular magnetic field $\boldsymbol{B} = \nabla \times \boldsymbol{A}$ is

$$H = (1/2m)|\boldsymbol{p} + e\boldsymbol{A}/c|^2 + V(\boldsymbol{r}). \tag{2}$$

When $B = 0$ and $V(\boldsymbol{r}) = 0$, the DOS per unit area for both spins is a constant $n(E) = n_0 =$

$m/\pi\hbar^2$. In magnetic field B with $V(\boldsymbol{r}) = 0$, the electrons are confined to LLs having energy $E_n = \hbar\omega(n + \frac{1}{2})$, where $\omega = eB/mc$ is the cyclotron frequency. The DOS is

$$n(E) = n_l \sum_{n=0}^{\infty} \delta(E - E_n) \tag{3}$$

where $n_l = n_0\hbar\omega = 1/\pi l^2$ is the density of electrons that fill a single LL. Here l is the cyclotron radius. In the presence of impurities, the potential $V(\boldsymbol{r})$ seen by the electrons fluctuates from point to point due to fluctuations in the impurity concentration. Thus, the total electron energy ($E = T + V(\boldsymbol{r})$) fluctuates and not all electrons have the same energy. This broadens the LLs.

We assume the impurities are equally likely to be anywhere in the electron plane. The variance of $V(\boldsymbol{r})$ is modelled by the Gaussian function (1). For example, if the disorder is due to impurities located at random points $\boldsymbol{R}_i$ in the plane,

$$V(\boldsymbol{r}) = \sum_i v(\boldsymbol{r} - \boldsymbol{R}_i)$$

then

$$W(\boldsymbol{r} - \boldsymbol{r}') = n \int \mathrm{d}\boldsymbol{R}\, v(\boldsymbol{r} - \boldsymbol{R}) v(\boldsymbol{r}' - \boldsymbol{R})$$

where n is the impurity concentration. Assuming a Gaussian potential, $v(\boldsymbol{r})$ leads directly to (1). We obtain $n(E)$ from the propagator for the electron averaged over the fluctuations in $V(\boldsymbol{r})$ using the path-integral method

$$K(\boldsymbol{r}, \boldsymbol{r}'; T) = \int D[\boldsymbol{r}(\tau)]\, \mathrm{e}^{(\mathrm{i}/\hbar)S} \tag{4}$$

where S is the averaged action corresponding to H and

$$\int D[\boldsymbol{r}(\tau)]$$

denotes the path integral with boundary conditions $\boldsymbol{r}(0) = \boldsymbol{r}$ and $\boldsymbol{r}(T) = \boldsymbol{r}'$. For a uniform system, $n(E)$ is related to a diagonal element of K by

$$n(E) = (A/\pi\hbar)\,\mathrm{Re} \int_0^{\infty} \mathrm{d}T\, K(0, 0; T)\, \mathrm{e}^{(\mathrm{i}/\hbar)ET}. \tag{5}$$

After completing the routine evaluation of the averaged propagator in the first cumulant approximation, we get

$$K(0, 0; T) = \left(\frac{m}{2\pi\mathrm{i}\hbar T}\right)\left(\frac{\omega T}{2\sin(\omega T/2)}\right) \exp\left[-\frac{\xi_L}{2\hbar^2} T \int_0^T \mathrm{d}y \left(\frac{1}{G(T, y)}\right)\right] \tag{6}$$

where

$$G(T, y) = 1 + \frac{8\mathrm{i}\sin[\omega(T - y)/2]\sin(\omega y/2)}{x\sin(\omega T/2)} \tag{7}$$

where ξ_L is the magnitude of the Gaussian variance, $x = \hbar\omega/E_L$, and $E_L = \hbar^2/2mL^2$. The DOS in (5) cannot be evaluated analytically due to the complicated part in the

exponential term of $K(0, 0; T)$. In [23] the DOS was considered by making a large-T approximation where (5) has an analytic form

$$n(E) = n_l(2\pi\Gamma^2)^{-1/2} \sum_{n=-0}^{\infty} \exp[-(E - E_n)^2/2\Gamma^2] \tag{8}$$

with

$$\Gamma^2 = \xi_L \frac{x}{4 + x} = \frac{\xi_L}{1 + 2l^2/L^2}. \tag{9}$$

These expressions are the same as that obtained by Gerhardts [22] for $n = 0$.

However, the width Γ of every LL is equal which seems to contradict the direct measurement of Kukushkin and Timofeev using radiative recombination spectra of 2D electrons in a MOS inversion layer [13]. It is certainly a result of using the long-time limit that we obtain a value Γ which is independent of the Landau index. To avoid such an approximation, numerical integration can be used to evaluate the integral in (5) exactly and then comparison with experiments can be done to justify our expression for $n(E)$. It is known that optical spectroscopy of 2D electrons makes it possible to study the entire $n(E)$ dependence and yield $n(E_F)$. In all other experiments, one determines only the thermodynamic DOS, i.e. the quantity dn/dE_F, which is in general not equal to $n(E_F)$. So it is more reasonable to compare our numerical results with the magneto-optic experiments.

To calculate the density of states from (5), (6), and (7), we write

$$n(E) = n_0(2/\pi) \sum_{n=0}^{\infty} \int_0^{\infty} dt \, \mathrm{Re} \exp\{2i[\nu/x - (n + \tfrac{1}{2})]t + f'(t)\} \tag{10}$$

where

$$f'(t) = \frac{-t \sin t}{2ix} \xi'_L \int_0^t dy \frac{1}{[(x/4i) \sin t - \cos t + \cos y]} \tag{11}$$

with $\xi'_L = \xi_L/E_L^2$ and $\nu = E/E_L$. The integration in (11) can be performed analytically. We take $t = \pi N + \theta\,(-\pi/2 \leqslant \theta \leqslant \pi/2)$, so that (10) becomes

$$n(E) = n_0 \sum_{n=0}^{\infty} K((\nu - x(n + \tfrac{1}{2}))), \tag{12}$$

where

$$K(\nu) = (2/\pi) \int_0^{\infty} dt \, \mathrm{Re} \exp(2i\nu t/x + f'(t)) \tag{13}$$

with

$$f'(t) = -(\pi N + \theta)\xi'_L \frac{\sin\theta}{2ix\sqrt{a^2 - 1}} \left[\pi N + 2 \tan^{-1}\left(\frac{\sqrt{a - 1}}{\sqrt{a + 1}} \tan(\theta/2)\right)\right] \tag{14}$$

and

$$a = (x/4i) \sin\theta - \cos\theta \qquad |a - \sqrt{a^2 - 1}| < 1. \tag{15}$$

The time integral is then performed by Gaussian quadrature. Figure 1 shows the DOS

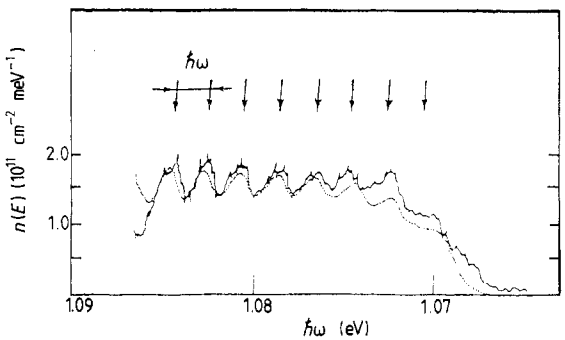

Figure 2. Emission spectrum found when the angle between the magnetic field and the normal to the plane of the 2D layer is 60° ($\hbar\omega = 2$ meV). The broken curve shows the numerical result for the DOS with $\xi_L = 6.8$ meV^2 and $L = 97$ Å.

and its energy dependence from the radiation spectra obtained from the MOS structure for $n = 2.7 \times 10^{12}$ cm^{-2} at $T = 1.6$ K in a magnetic field $B = 0$ (figure 1, spectrum 1) and $B = 7$ T perpendicular to the 2D layer (figure 1, spectrum 2, $v = 16$, $N \leqslant 4$). From our expression in (12), by choosing an appropriate $\xi_L = 6.8$ meV^2 with $L = 97$ Å corresponding to $E_L = \hbar^2/2m^*L^2 = 2$ meV (using $m^* = 0.2m_e$ as in the experiments), our numerical results for the DOS can fit the experiments very well as shown in figure 1 (broken curve). Furthermore, when the magnetic field B makes an angle 60° with the normal, our results are still in good agreement as shown in figure 2 (broken curve). We find that the width Γ tends to decrease while the Landau index N increases. The DOS between LLs is not exponentially small and is an appreciable fraction of $n(E)$ at $B = 0$. Although our results for the DOS between LLs seem to be a little lower than those obtained in experiments by the magneto-optic method, they are acceptable because of the tendency of peak-height increasing and peak-width decreasing as the Landau index increases, and are in good agreement.

In summary, we are able to reproduce the density of states observed in a typical two-dimensional electron gas, such as that shown in figure 1 and figure 2, using a simple model of disorder having Gaussian variance ξ_L with finite correlation length $L \sim 100$ Å.

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